(1) $D_i = 0.018\,m$, $D_o = 0.021\,m$ → $A = \pi D L$

$h_i = 210\ W/m^2K$, $h_o = 70\ W/m^2K$
(steam) (air)

$k_{Brass} \approx 111\ W/mK$ (this may vary, depending on your temp. and source)

$$R_{tot} = \frac{1}{h_i A_i} + \frac{1}{2\pi k L}\ln\frac{r_o}{r_i} + \frac{1}{h_o A_o}$$

$$R = \frac{1}{UA} \rightarrow U_o = \frac{1}{R A_o},\quad U_i = \frac{1}{R A_i}\quad \left(\text{or } U_o = U_i \frac{A_i}{A_o}\right)$$

a) $$U_i = \frac{1}{\frac{1}{h_i} + \frac{\pi D_i L}{2\pi k L}\ln\frac{D_o/2}{D_i/2} + \frac{\pi D_i L}{h_o \pi D_o L}} = \frac{1}{\frac{1}{h_i} + \frac{D_i}{2k}\ln\frac{D_o}{D_i} + \frac{D_i}{h_o D_o}}$$

$$= \frac{1}{\frac{1}{210} + \frac{0.018}{111\times 2}\ln\frac{0.021}{0.018} + \frac{0.018}{(70)(0.021)}} = \frac{1}{0.017}\ \frac{W}{m^2K}$$

$U_i = 58.76\ W/m^2 \cdot K$

b) $$U_o = U_i \frac{\pi D_i L}{\pi D_o L} = U_i \frac{D_i}{D_o} = \boxed{50.36\ W/m^2K}$$

c) if $R_f = 0.00018\ m^2K/W$, $R_{tot} = \frac{1}{h_i A_i} + \frac{R_{fi}}{A_i} + \cdots$, and

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{D_i}{2k}\ln\frac{D_o}{D_i} + \frac{D_i}{h_o D_o} + R_{fi}} = \frac{1}{0.0172}\ \frac{W}{m^2K}$$

$U_i = 58.14\ \frac{W}{m^2K}$

not a big change! It makes sense that fouling leads to a smaller U → think about <u>why</u>.

(2) **Water**

$\dot{m} = 12\ kg/s$

$T_i = 40°C$

$C_p = 1059\ J/kgK$ ⟷ $C_p = 4178\ J/kgK$ (might vary a bit)

Air

$\dot{m} = 2\ kg/s$

$T_i = 460°C$

Hxgr

$A = 14\ m^2$

$U = 275\ \frac{W}{m^2K}$

Should use ε-NTU method.

First, find C_r and NTU

$(\dot{m}C_p)_{water} = 50136\ W/K$

$(\dot{m}C_p)_{air} = 2118\ W/K = C_{min}$

$C_r = \frac{C_{min}}{C_{max}} = 0.0422$

$NTU = \frac{UA}{C_{min}} = \frac{275 \times 14}{2118} = 1.8178$

a) From the table on-line

$$\varepsilon_{parallel} = \frac{1 - \exp(-NTU(1+C_r))}{1+C_r} = \boxed{0.815}$$

$$\varepsilon_{\substack{unmixed\\cross}} = 1 - \exp\left\{\frac{NTU^{0.22}}{C_r}\left[\exp(-C_r NTU^{0.78}) - 1\right]\right\} = \boxed{0.828}$$

b) $\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{\dot{Q}}{C_{min}(T_{hi} - T_{ci})}$ ⟵ $= 889.56\ kW$

$$\boxed{\begin{aligned}\dot{Q}_{parallel} &= 725.2\ kW\\ \dot{Q}_{cross} &= 736.2\ kW\end{aligned}}$$

c) $\dot{Q} = C_w(T_{w,o} - T_{w,i})$

$T_{wo} = \frac{\dot{Q}}{C_w} + T_{wi}$

$\dot{Q} = C_{air}(T_{air,i} - T_{air,o})$

$T_{air,o} = T_{air,i} - \frac{\dot{Q}}{C_{air}}$

$T_{wo,parallel} = 54.46°C$	$T_{air,o,parallel} = 117.6°C$
$T_{wo,cross} = 54.68°C$	$T_{air,o,cross} = 112.4°C$

(3)

Water

$\dot{m}_w = 10{,}000 \frac{lbm}{hr}$

$T_{w_i} = 60°F$

$c_p \approx 1 \frac{Btu}{lbm\,°R}$

Oil

$\dot{m}_{oil} = 20{,}000 \frac{lbm}{hr}$

$T_{oil,i} = 200°F$

$T_{oil,out} = 150°F$

$c_p = 0.5 \frac{Btu}{lbm\,°R}$

$U = 50 \frac{Btu}{hr \cdot ft^2 \cdot °F}$

$A = ?$

$C_w = (\dot{m}c_p)_w = 10{,}000 \frac{Btu}{hr\,°R}$

$C_{oil}\ (\dot{m}c_p)_{oil} = 10{,}000$ "

$\rightarrow$ so $C_w = C_{oil}$

Recall from LMTD, when $C_{cold} = C_{hot}$, $\Delta T_1 = \Delta T_2 = \Delta T_{\text{everywhere}}$

so, $\dot{Q} = UA\Delta T$. Also $\dot{Q} = \dot{m}_{oil} c_{p_{oil}} (T_{oil,in} - T_{oil,out})$

$\dot{Q} = 10000 \frac{Btu}{hr\,°R} 50°R = 500{,}000 \frac{Btu}{hr}$

$$A = \frac{500{,}000\ Btu/hr}{50 \frac{Btu}{hr\,ft^2\,°F}\underbrace{(T_{oil,out} - T_{w,in})}_{\text{B/c Counterflow}}} = \boxed{111.1\ ft^2}$$

(= 90°)

(4) Follow similar steps as in-class example

(A) **Water (pipe)**

$\dot{m}_w = 5000 \frac{lbm}{hr}$

$\rho_w = 61.3 \frac{lbm}{ft^3}$

$c_p = 1 \frac{Btu}{lbm\,°R}$

$\nu = 4.81 \times 10^{-6} \frac{ft^2}{s}$

$\alpha = 0.00618 \frac{ft^2}{hr}$

$k = 0.3787 \frac{Btu}{hr\,ft\,°R}$

$P_r = 2.8$

$T_{w,i} = 150°F$

Oil (annulus)

$\dot{m}_{oil} = 400 \frac{lbm}{hr}$

$\rho = 54.8 \frac{lbm}{ft^3}$

$c_p = 0.464 \frac{Btu}{lbm\,°R}$

$\nu = 4.27 \times 10^{-3} \frac{ft^2}{s}$

$\alpha = 0.00327 \frac{ft^2}{hr}$

$k = 0.0832 \frac{Btu}{hr\,ft\,°R}$

$P_r = 4699$

$T_{oil,in} = 90°F$

$T_{oil,out} = 100°F$

(B) **Tubing** $k = 231 \frac{Btu}{hr\,ft\,°R}$

Counterflow

$1\frac{1}{4}$: ID = 0.1076'

OD = 0.1146'

2: ID = 0.1674'

L = 15'

(C) $A_{pipe} = \frac{\pi ID_p^2}{4} = 0.00909\,ft^2$

$A_{ann} = \frac{\pi}{4}(ID_a^2 - OD_p^2)$

$A_{annulus} = 0.0117\,ft^2$

(D) $V_{pipe} = \frac{\dot{m}_w}{\rho_w A_{pipe}} = \frac{5000 \frac{lbm}{hr}}{(61.3 \frac{lbm}{ft^3})(0.009 ft^2)} = 2.49 \frac{ft}{s}$

$V_{ann} = \frac{\dot{m}_{oil}}{\rho_{oil} A_{ann}} = 0.173\,ft/s$

(E) $Re_{pipe} = \frac{V_{pipe} ID_{pipe}}{\nu_w} = \boxed{55{,}782 = Re_{water}}$

$Re_{ann} = \frac{V_{ann} D_{equiv}}{\nu_{oil}}$, $D_{equiv} = \frac{ID_{ann}^2 - OD_{pipe}^2}{OD_{pipe}} = 0.13\,ft$

$Re_{ann} = \boxed{5 = Re_{oil}}$

— Part a

(F) Nusselt #s

Water: Turbulent flow, Pr in correct range

$Nu = 0.023\,(Re^{0.8})\,Pr^{0.3}$

(4, cont.)

$$Nu_{water} = 0.023(55782^{0.8})(2.8^{0.3}) = \boxed{196.37}$$

Oil: Laminar

$$Nu_{oil} = 1.86\left(\frac{D_{eq} Re\, Pr}{L}\right)^{\frac{1}{3}} = 1.86\left(\frac{(0.13)(5)(4699)}{15}\right)^{1/3} = \boxed{10.94}$$

← Part b

(G) $$h_w = h_{pipe} = \frac{Nu_p \cdot k_w}{ID_p} = \frac{196.37 \cdot 0.3787}{0.1076} = 691.13 \frac{Btu}{hr\, ft^2 {}^\circ R}$$

$$h_a = \frac{Nu_a\, k_{oil}}{D_{equiv}} = 7 \frac{Btu}{hr\, ft^2 {}^\circ R}$$

(H) Overall Hxfr Coeff.

$$\frac{1}{U_o} = \frac{OD_p}{ID_p\, h_p} + \frac{OD_p}{2\, k_{copper}} \ln\frac{OD_p}{ID_p} + \frac{1}{h_a}$$

$$= \frac{0.1146}{0.1076 \cdot 691.13} + \frac{0.1146}{2 \times 231} \ln\frac{0.1146}{0.1076} + \frac{1}{7} = 0.1444$$

$$\boxed{U_o = 6.925 \frac{Btu}{hr\, ft^2 {}^\circ R}}$$ ← Part c (Think about what is driving this # ...)

(I)

$$\dot{Q} = U_o A_o (LMTD) \text{ or } fn(\varepsilon - NTU)$$

ε-NTU is easier: $C_w = 5000 \times 1 = 5000 \frac{Btu}{hr\, {}^\circ R}$

$C_{oil} = 400 \times 0.464 = 185.6 \frac{Btu}{hr\, {}^\circ R} = C_{min}$

$C_r = 0.037$

$$NTU = \frac{U_o A_o}{C_{min}} = \frac{(6.925)(\pi (0.1146)15)}{185.6} = 0.2015$$

$$\varepsilon = \frac{1 - \exp(-NTU(1-C_r))}{1 - C_r \exp(-NTU(1-C_r))} = 0.1849$$

Part d ↓

$$\dot{Q} = \varepsilon \dot{Q}_{max} = \varepsilon C_{min}(T_{hi} - T_{ci}) = 0.1849(185.6)(60) = 2059 \frac{Btu}{hr} = 0.57 \frac{Btu}{s}$$