ME 1065 – HW 4 Solutions

1.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f + h_m - h_{pump}$$

$$P_1 = P_2\ ,\ V_1 = V_2 \approx 0\ ,\ z_2 - z_1 = 10\ m$$

$$h_f = f\,\frac{L}{d}\,\frac{V^2}{2g}\ ,\ h_m = K\,\frac{V^2}{2g}$$

a) For 25-mm pipe, $K = \underbrace{0.5}_{\text{entrance}} + \underbrace{1.5}_{\substack{90^\circ \\ \text{reg} \\ \text{screwed}}} + \underbrace{0.43}_{\substack{90^\circ \\ \text{reg} \\ \text{flanged}}} + \underbrace{1.0}_{\text{exit}} = 3.43$, $\varepsilon = 0.15\ mm$

Volume of tank $= 1.5 \times 1.5 \times 1.2 = 2.7\ m^3$

Time $= 3600\ s$

Flow rate $= \frac{\forall}{t} = \frac{2.7\ m^3}{3600\ s} = 0.00075\ \frac{m^3}{s} = Q$

$$V = \frac{Q}{A} = \frac{Q}{\pi\left(\frac{0.025\,m}{2}\right)^2} = 1.528\ m/s$$

$$Re = \frac{\rho V D}{\cancel{\mu}} = \frac{(1.528\ \frac{m}{s})(0.025\,m)}{87.5\times10^{-8}\ m^2/s} = 43654\ \left(4.4\times10^4\right),\ \frac{\varepsilon}{D} = \frac{0.15}{25} = 0.006$$

From Moody, $f = 0.034$

$$h_{pump} = (10\ m) + \underbrace{0.034}_{f}\ \underbrace{\frac{40\ m}{0.025\ m}}_{L/d}\ \underbrace{\frac{\left(1.528\ \frac{m}{s}\right)^2}{2\,(9.8\ m/s^2)}}_{V^2/2g} + \underbrace{3.43}_{K}\ \underbrace{\left(\quad\right)}_{V^2/2g} = 16.89\ m$$

$$\dot{W}_{water} = \rho g Q h_p = (997)(9.8)(0.00075)(16.89) = 123.75\ W$$

$$\dot{W}_{supplied} = \frac{123.75}{0.7} = \boxed{176.8\ W}$$

b) Now, $\dot{W}_{supplied} = 176.8 \times 1.5 = 265.2\ W$

$\dot{W}_w = 185.6\ W$

Both h_p + Q are unknown, since new $\dot{W}$ will affect the flow velocity.

$$185.6 = \underbrace{(997)}_{\rho}\underbrace{(9.8)}_{g}\underbrace{\left(\pi\left(\frac{0.025}{2}\right)^2\right)}_{A} V \left[10 + f\,\frac{40}{0.025}\,\frac{V^2}{(2)(9.8)} + 3.43\,\frac{V^2}{2(9.8)}\right]$$

Also a fn of V through the Reynolds number, so it looks like an iterative solution, or can solve in a computer program

To solve iteratively:

- Guess V
- Find Re
- Calculate / Lookup f (It's likely this will be turbulent) $\rightarrow \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$

1b, cont.

- Solve right hand side + check if it = LHS
- If not, guess new V

Answer: $V = 1.892\ \text{m/s}$, $f = 0.0336$, $Re = 5.41\times10^4$ $(h_p = 20.4)$

so $Q = AV = 0.000929\ \text{m}^3/\text{s}$

$$\text{Time} = \frac{\forall}{Q} = \boxed{2907\ \text{s} = 48.45\ \text{min}}$$

c) Now $d = 40$ mm, $\dot{W}_w = 123.75$ W

$$123.75 = \underset{\rho}{(997)}\underset{g}{(9.8)}\underbrace{\pi\left(\frac{0.04}{2}\right)^2}_{A} V\left[10 + f\frac{40}{0.04}\frac{V^2}{(2)(9.8)} + K_{new}\frac{V^2}{2(9.8)}\right]$$

a function of both the new V and new ε/D

$$\varepsilon/D = \frac{0.15}{40} = 0.00375$$

new values for D=40mm

$$K_{new} = 0.5 + \underset{\substack{90^\circ\\ \text{scr.}}}{1.2} + \underset{\substack{90^\circ\\ \text{fl.}}}{0.4} + 1.0 = 3.1$$

Again, this can be solved iteratively, or by computer:

$V = 0.888\ \text{m/s}$, $f = 0.0305$, $Re = 4.06\times10^4$, $h_p = 11.35\text{m}$

so $Q = 0.001116\ \text{m}^3/\text{s}$, and

Time $= 2420\text{s} = 40.3$ min

2.

a) Looking at how Q + h vary with diameter:

28" → 32"	28" → 36.75"	On a given pt. on the 28" curve:
$Q_1 = \left(\frac{D_1}{D_2}\right)^3 Q_2 = 1.493\, Q_2$	$Q_1 = 2.261\, Q_2$	$Q = 8000$ gal/min
$h_1 = \left(\frac{D_1}{D_2}\right)^2 h_2 = 1.306\, h_2$	$h_1 = 1.723\, h_2$	$h = 350$ ft

So, are $Q = 11{,}944$; $h = 457.1$ on the 32" curve, and $Q = 18{,}088$; $h = 603.05$ on the 36.75' curve?

$\boxed{\text{Roughly, yes.}}$

b) i) $C_Q = \frac{Q}{\omega D^3} \rightarrow C_{Q_1} = C_{Q_2} \rightarrow \frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3} \rightarrow \frac{Q_1}{Q_2} = \left(\frac{\cancel{\omega_1}}{\cancel{\omega_2}}\right)^{1}\left(\frac{D_1}{D_2}\right)^3$ ✓

ii) $C_H = \frac{gh}{\omega^2 D^2} \rightarrow C_{H_1} = C_{H_2} \rightarrow \frac{g h_1}{\omega_1^2 D_1^2} = \frac{g h_2}{\omega_2^2 D_2^2} \rightarrow \frac{h_1}{h_2} = \left(\frac{\cancel{\omega_1}}{\cancel{\omega_2}}\right)^2\left(\frac{D_1}{D_2}\right)^2$ ✓

3.

Given the tabular data, and property data, we know

$Q,\ H,\ bhp\ (P),\ \omega,\ \rho,\ D$

$\rho = 998\ kg/m^3$, $D = 0.37\ m$

$\omega = 2140 \frac{rev}{min} \cdot \frac{1\ min}{60\ s} \cdot \frac{2\pi\ rad}{1\ rev} = 224.1 \frac{rad}{s}$

so, at each point, we can calculate

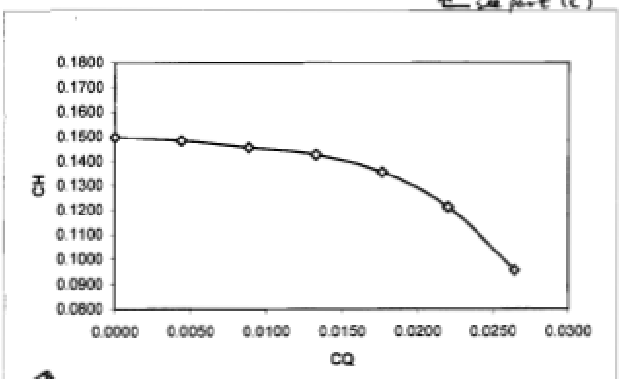
$C_H = gH/\omega^2 D^2, \quad C_Q = Q/\omega D^3, \quad C_P = bhp/\rho\omega^3 D^5$ (Note: This is in W, not kW)

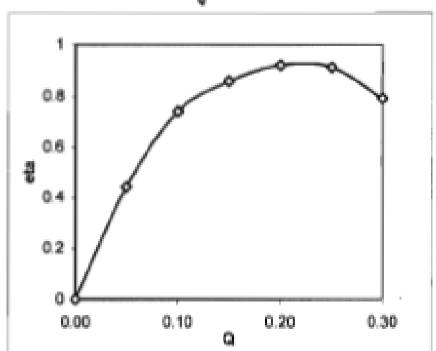
and $\eta = \frac{\rho g Q H}{bhp}$ (again, in W). I did this in Excel, as shown:

Q, m3/s	0.00	0.05	0.10	0.15	0.20	0.25	0.30
H, m	105	104	102	100	95	85	67
P, kW	100	115	135	171	202	228	249
CH	0.1497	0.1482	0.1454	0.1425	0.1354	0.1212	0.0955
CQ	0.0000	0.0044	0.0088	0.0132	0.0176	0.0220	0.0264
CP	0.0013	0.0015	0.0017	0.0022	0.0026	0.0029	0.0032
eta	0	0.442	0.739	0.858	0.920	0.912	0.790

see part (c)

(Part a)
It appears the best η is approx. 0.92 @ $Q \approx 0.22$





Part (b) → This looks as expected from Fig 4b of the handout

(c) Now, want $7000 \frac{gal}{min} \cdot \frac{3.7854\times10^{-3}\ m^3}{1\ gal} \cdot \frac{1\ min}{60\ s} = 0.4416 \frac{m^3}{s}$, Power = 400,000 W

Kerosene @ 20°C: $\rho = 804\ kg/m^3$, choosing a base case Q_1 of $0.15\ m^3/s$,

we know $C_{Q_1} = C_{Q_2}$ and $C_{P_1} = C_{P_2}$, so

$$\omega_1 D_1^3 \frac{Q_2}{Q_1} = \omega_2 D_2^3 \rightarrow (224.1)(.37^3)\left(\frac{0.4416}{0.15}\right) = \omega_2 D_2^3$$

$$\omega_1^3 D_1^5 \frac{\rho_1 BHP_2}{\rho_2 BHP_1} = \omega_2^3 D_2^5 \rightarrow (224.1)^3(0.37^5)\frac{998}{804}\left(\frac{400}{171}\right) = \omega_2^3 D_2^5$$

Solving Simultaneously:

$$\boxed{\omega_2 = 129.3 \frac{rad}{s} = 1234\ rev/min \qquad D_2 = 0.637\ m = 63.7\ cm}$$